

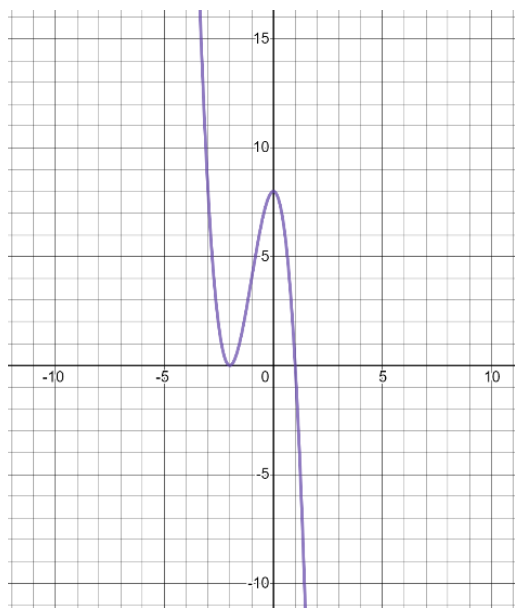
MATH 115 Final Exam - Summer 2020

This is an open book exam. You may refer to the textbook and other course materials and may use a calculator. You must complete the exam as an individual. Neither collaboration nor consultation with others is allowed.

Record your answers and work on the separate answer sheet provided. Simplify your answers. Unless the accuracy level is noted in the problem (specifying the number of decimal places required), all answers must be exact values.

Problems 1 to 9: short answer, with no work required. Problems 10 to 25: work is required.

1. 2 pts. Use a calculator to find the function value accurate to four decimal places: $\sin(19\pi/16)$
2. 2 pts. A can of soda at 80°F is placed in a refrigerator that maintains a constant temperature of 37°F . The temperature T of the soda t minutes after it is placed in the refrigerator is given by $T(t) = 37 + 43e^{-0.058t}$. When will the soda be 60°F ? (Round to the nearest tenth of a minute.)
3. 2 pts. Find a polynomial function $f(x)$ that satisfies all of the following characteristics:
Degree of the polynomial is 4;
There is an x intercept at -3 ; and
As x approaches ∞ , $f(x)$ approaches $-\infty$; and as x approaches $-\infty$, $f(x)$ approaches $-\infty$.
4. 2 pts. Find the exact value of $\cot[\arcsin(-3/7)]$
5. 3 pts. Find a sinusoidal function with the following characteristics: amplitude=1, period= 4π , and phase shift = $-\pi/6$.
6. 6 pts. See the function $f(x)$ graphed below. Write your answers in interval notation, where applicable.
6A. Determine the interval(s) on which the function is increasing.
6B. Solve $f(x) \leq 0$
6C. Is the function even, odd, or neither?



7. 3 pts. $\sec\theta = -1.3456$, for an angle θ between 180° and 270° . Use a calculator to find θ , rounding to the nearest degree.

8. 2 pts. Let $f(x) = \frac{2-x}{5x^2 - 6x + 1}$ and $g(x) = 1 - 2x$.
Find the composite function $f \circ g(1)$ and simplify.

9. 4 pts. Points $(-5, 1)$ and $(3, 5)$ are endpoints of the diameter of a circle.
A. What is the exact length of the diameter? (Simplify as much as possible)
B. What is the equation of the circle?

Work is required to be shown for Problems 10 to 25.

10. 5 pts. SHOW WORK. Use exact values and interval notation.
For the function $f(x) = |5 - 3x| + 2$
A. Find the minimum point of the function
B. Solve $f(x) \leq 6$.

11. 4 pts. Solve, find exact answers in the interval $[0, 2\pi)$. SHOW WORK. $\tan\left(2x + \frac{\pi}{6}\right) = -\sqrt{3}$

12. 4 pts. SHOW WORK. Vector v is $\langle -10, 12 \rangle$ and vector w is $\langle 4, -3 \rangle$.
12A. Find the dot product $v \cdot w$. (Exact value)
12B. Find the angle between the vectors v and w , accurate to the nearest hundredth of a degree.

13. 4 pts. Solve the trigonometric equation $\sqrt{2}(\cos x)(\sin x) + \sin x = 0$ in the interval $[0, 360^\circ)$.
(Answers should be exact values, measured in degrees.) SHOW WORK.

14. 9 pts. $\tan\alpha = 3$ for α an angle between π and $3\pi/2$. Find exact values for the following. Simplify your answers. SHOW WORK.
14A. $\sec(-\alpha)$
14B. $\tan(2\alpha)$
14C. $\sin(\pi/2 - \alpha)$

15. 3 pts. Find the equation for a line which passes through the points $(60, -100)$ and $(-20, 1000)$. Write the equation in slope-intercept form. SHOW WORK.

16. 3 pts. SHOW WORK. Solve with accuracy up to 4 decimal places. $(\frac{1}{2})^{x+2} = 9$

17. 4 pts. Prove the identity. Work only one side of the equation to get it to look like the other side. Provide an explanation for each step of your work. Be specific in your explanations. ("Simplify" is not a specific explanation.)
(4 points = algebraically correct proof, with all steps included, and explanations provided for each step.
2 points = Proof missing steps, but algebraically correct and explanations provided for each step.
0 points = Algebra is not correct and/or explanations are not provided for each step.)

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{\sin\theta - \tan\theta}{-2\tan\theta}$$

18. 4 pts. Find the equation, in standard form, of a parabola with directrix at $x = 1$ and focus at $(-3, 1)$. Answers should include exact values and not approximations. SHOW WORK.

19. 4 pts. $a=12$, $b= 24$ $c = 16$ in a triangle where angle A is opposite side a, angle B is opposite side b, and angle C is opposite side c.

A. Solve the triangle(s). Round your final answers to the nearest tenth. SHOW WORK.

B. Find the area. Round your final answers to the nearest tenth. SHOW WORK.

20. 6 pts. SHOW WORK. For the function $g(x) = \frac{1-9x^2}{x^2-4x}$

20A. Find the vertical asymptote(s).

20B. Solve $g(x) \leq 0$. Use interval notation.

20C. Find the end behavior of the function.

21. 4 pts Find the equation of the hyperbola that has vertices at $(-4, 2)$ and $(8, 2)$ and a focus at $(-7, 2)$. SHOW WORK.

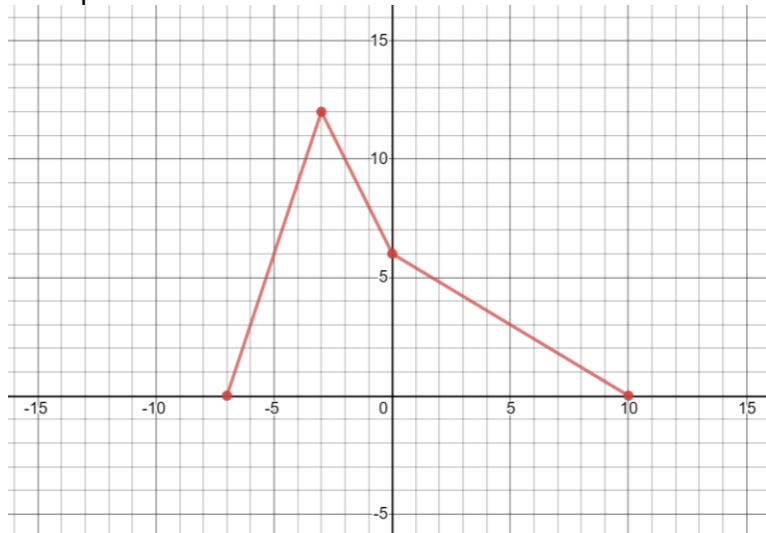
22. 4 pts. SHOW WORK. A firework is launched from a height of 2 feet with an initial velocity of 120 feet per second. The height h of the firework (measured in feet) at t seconds after launch is given by $h(t) = -16t^2 + 120t + 2$.

22A. How many seconds after launch does the firework attain maximum height? (exact answer)

22B. What is the maximum height? (exact answer)

23. 4 pts. Solve $\frac{x-5}{x-4} + \frac{8}{x^2-16} = 0$ SHOW WORK.

24. 7 pts. Use exact values and interval notation. SHOW WORK. For the function $f(x)$ pictured below:



24A. Find the domain and range

24B. Solve $f(x) \leq 3$

24C. Sketch the graph of $y = -f(x + 4) + 3$

25. 5 pts. Use exact values. SHOW WORK. For the equation $9y^2 + 10x^2 - 40x = 50$

25A. Find the center.

25B. Find the foci.

25C. Find the endpoints of the minor axis.